

1995 MATHEMATICS OLYMPIAD LECTURE NOTES

Vectors

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For our purposes, a *vector* is essentially a *directed arrow*. A *vector* has the properties of *length* and *direction*, but **not** *position*. Thus, we say two vectors are *equal* if they have the **same** *length* and **same** *direction*.

If A, B are two points then \overrightarrow{AB} is the vector that joins A and B ; it has *length* equal to the length of the line segment that joins A and B , and *direction* parallel to an arrow pointing from A to B . We assign coordinates to \overrightarrow{AB} by subtracting the respective coordinates of A from those of B . The *position vector* of a point A is the vector \overrightarrow{OA} where O is the origin, relative to which, A has coordinates. (So A and \overrightarrow{OA} expressed as coordinates look the same.)

Example.

Suppose $A = (1, 3)$ and $B = (2, 7)$ then

$$\overrightarrow{AB} = B - A = (2, 7) - (1, 3) = (2 - 1, 7 - 3) = (1, 4).$$

Suppose and $C = (5, 4)$ and $D = (6, 8)$ then

$$\overrightarrow{CD} = D - C = (6, 8) - (5, 4) = (6 - 5, 8 - 4) = (1, 4).$$

Observe that \overrightarrow{AB} and \overrightarrow{CD} are *equal* despite the fact they connect different pairs of points in space.